DESIGNS RELATED TO GRAPHS

(MRP(S)-0154/12-13/KAMY008/UGC-SWRO, Dated:23-09-2013)

A MINOR RESEARCH PROJECT SUBMITTED

TO

UNIVERSITY GRANTS COMMISSION

South Western Regional Office

PK Block, Palace Road, Gandhinagar,

Banglore- 560009

BY

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CERTIFICATE

This is to certify that the minor research project entitled "**DESIGNS RELATED TO GRAPHS**" (**MRP(S)-0154/12-13/KAMY008/UGC-SWRO, Dated: 23-09-2013**) submitted by **Dr. M P SUMATHI**, Assistant Professor, Department of Mathematics, SBRR Mahajana First Grade College, Mysore - 12, to University Grants Commission, South Western Regional Office, PK Block, Palace Road, Gandhinagar, Banglore- 560009, is a genuine record of research work carried out by her in SBRR Mahajana First Grade College, Mysore - 12, during the 11th plan period.

I further certify that the matter embodied in this project is original and carried out accordingly to the plan proposal and guidelines of the University Grants Commission and not been submitted by her to this or any other universities for the award of any degree, diploma or fellowship.

Place: Mysore **(Dr. Venkataramu S)**
Date: **(Principal)**

DECLARATION

I **Dr. M P SUMATHI**, Assistant Professor, Department of Mathematics, SBRR Mahajana First Grade College, Mysore - 12, hereby declare that the Minor Research Project Report entitled "**DESIGNS RELATED TO GRAPHS**"
(**MRP(S)-0154/12-13/KAMY008/UGC-SWRO, Dated: 23-09-2013**)
is submitted by me to University Grants Commission, South Western Regional Office, PK Block, Palace Road, Gandhinagar, Banglore- 560009 during the 11th plan period is the result of the bonafied research work.

I further declare that the results here presented are original and carried out accordingly to the plan proposal and guidelines of the University Grants Commission.

Place: Mysore (**Dr. M P Sumathi**)
Date:

Executive Summary

The origin of graph theory can be traced back to Euler's work on the Konigsberg bridges problem (1735), which subsequently led to the concept of an Eulerian graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A. F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem. Then in 1856, Thomas. P. Kirkman and William R. Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H. Dudeney mentioned a puzzle problem. Even though the four color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of Graph Theory.

Caley studied particular analytical forms from differential calculus to study the trees. This had many implications in theoretical chemistry. This lead to the invention of enumerative graph theory. Any how the term Graph was introduced by Sylvester in 1878 where he drew an analogy between Quantic invariants and covariant of algebra and molecular diagrams. In 1941, Ramsey worked on colorations which lead to the identification of another branch of graph theory called extremal graph theory. In 1969, the four color problem was solved using computers by Heinrich. The study of asymptotic graph connectivity gave rise to random graph theory.

Among the various concepts of graph theory, the concept of domination has existed for a long time. The concept of domination in graphs originated in 1850 with problem of placing minimum number of queens on a $n \times n$ chess board so as to cover or dominate every square. The problem of dominating the squares of chess board can be stated more generally as a problem of dominating the vertices of a graph.

Design theory is a study of combinatorial designs, which are collections of subsets with certain intersection properties. Block designs are combinatorial designs of a special type. This area is one of the oldest parts of combinatorics, such as in Kirkman's schoolgirl problem proposed in 1850. The solution of the problem is a special case of a Steiner system, which systems play an important role in the classification of finite simple groups. The area has further connections to coding theory and geometric combinatorics.

Design theory is an area of combinatorics which found rich application of algebraic structures. Combinatorial designs are generalization of finite geometries. Probably the history of Design Theory begins with the 1847 paper of Reverand T.P. Kirkman"On a problem of Combinatorics", Cambridge Dubluin Math.Journal. The great Statistician R.A. Fisher reinvented the concept of Combinatorial 2-design in the 20th century. Extensive application of algebraic structures for construction of 2-design (balanced incomplete block design) can be found in R.C. Bose 1939 Annals of Eugenics paper "On the construction of balanced incomplete block designs". Theory of 2-designs for $t \geq 2$ was in a state of rapid development . The 1987-88 Applied Combinatorics Program of IMA decided to devote the period from May1st 1988 to june 25th 1988 to concentrate on Design Theory to exchange ideas on lateast developments.Statisticians often abbreviate an incomplete-block design i9s balanced if it is partially balanced with respect to the trivial association scheme. On the other hand pure Mathematicians often Know BIBDs as 2-designs. Of course, a given incomplete -block design may be partially balanced with respect to more than one association scheme.

Introduction to Graph Theory and Designs and basic definitions which will be used in the subsequent chapters are collected in chapter one.

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Many authors have studied PBIBD with m-association scheme which are arising from some dominating sets of some graphs. H.B. Walikar, H.S.Ramane B.D.Acharya, H.S.Shekhareppa and S.Armugum have studied PBIBD arising from minimum dominating sets of paths and cycles, Anwar and Soner have studied Partial balanced incomplete block designs arising from some minimal dominating sets of SRNT graphs and Sumathi. M.P and N. D. Soner have studied Association scheme on some cycles related with minimum neighbourhood. Any undefined terms and notation, reader may refer to F.Harary. We concern here to study PBIBD and the association scheme which can be obtained from the minimum dominating sets in some certain $(C_n \circ K_1)$, $(C_n \circ K_2)$ graph , then we generalize the graph $(H \circ K_n)$ and it is open area to study the same things for the other graphs.The survey articles which helped us to complete our project has been highlighted in chapter two.

In chapter three,we are concerned with topic involving cycles and corona graphs with minimum dominating sets. We started discussing about $(C_n \circ K_1)$,$(C_n \circ K_2)$, $(C_n \circ K_3)$,$(C_n \circ K_n)$ graph and finally we determine the number of minimum dominating sets of graphs and prove that the set of all minimum dominating sets of $G = C_n \circ K_3$ forms a partially balanced incomplete block design with two association scheme. We have also generalized the results to $G \circ K_m$. We were able to extend our work to get many generalized results such as "Let H be a graph with n vertices and with $H \cong G \circ K_m$. Then the number of minimum dominating sets of G is $(m+1)^n$".

Theorem : For any graph $H \cong G \circ K_m$, we can get PBIBD with parameters, $(v = mn, k = m, r = (m+1)^{n-1}, b = (m+1)^n, \lambda_1 = 0, \lambda_2 = (m+1)^{n-2})$ and association scheme of 2-classes with :

$$P_1 = \begin{bmatrix} p^1_{11} & p^1_{12} \\ p^1_{21} & p^1_{22} \end{bmatrix} = \begin{bmatrix} (m-1) & 0 \\ 0 & (m+1)(n-1) \end{bmatrix}$$

and $P_2 = \begin{bmatrix} p^2_{11} & p^2_{12} \\ p^2_{21} & p^2_{22} \end{bmatrix} = \begin{bmatrix} 0 & m \\ m & (m+1)(n-2) \end{bmatrix}$.